

Find the 5<sup>th</sup> term of the sequence defined recursively by  $a_1 = 2$ ,  $a_2 = 3$ ,  $a_{n+2} = (n+1)a_n - 2a_{n+1}$  for  $n \geq 1$ . SCORE: \_\_\_\_\_ / 4 PTS

$$n=1: a_3 = 2a_1 - 2a_2 = \underline{2(2) - 2(3)} = -2 \quad (1)$$

$$n=2: a_4 = 3a_2 - 2a_3 = \underline{3(3) - 2(-2)} = 13 \quad (1\frac{1}{2})$$

$$n=3: a_5 = 4a_3 - 2a_4 = \underline{4(-2) - 2(13)} = -34 \quad (1\frac{1}{2})$$

Find the sum of the first 12 terms of the series  $\frac{10}{27} - \frac{2}{3} + \frac{6}{5} - \frac{54}{25} + \dots$ .

SCORE: \_\_\_\_\_ / 4 PTS

GEOMETRIC SERIES  $r = \frac{-\frac{2}{3}}{\frac{10}{27}} = -\frac{2}{3} \cdot \frac{27}{10} = -\frac{9}{5}$  ①

$$S_{12} = \frac{\frac{10}{27}(1 - (-\frac{9}{5})^{12})}{1 - (-\frac{9}{5})} = -152.8877 \quad \text{②}$$

Find the value of  $\sum_{i=4}^7 (-1)^{i-1}(i^2 - (i-2)!)$ .

SCORE: \_\_\_\_ / 5 PTS

$$\begin{aligned}& (-1)^3(4^2 - 2!) + (-1)^4(5^2 - 3!) + (-1)^5(6^2 - 4!) + (-1)^6(7^2 - 5!) \\&= -(16-2) + (25-6) - (36-24) + (49-120) \\&= \boxed{-14} \quad \boxed{+19} \quad \boxed{-12} \quad \boxed{-71} \\&= \boxed{-78} \quad \textcircled{1}\end{aligned}$$

You begin your first job, and taking the advice of your math instructor, you start saving for retirement immediately. SCORE: \_\_\_\_\_ / 3 PTS

Your first month, you deposit \$103 into your account. Each month after that, you deposit \$11 more than you deposited the previous month.

You leave the job the day before your 3<sup>rd</sup> anniversary. How much money did you deposit into the retirement account altogether ?

$$103 + (103+11) + (103+11+11) + \dots$$

ARITHMETIC SERIES  $d=11$  ①

$$S_{36} = \frac{36}{2} (103 + (103 + (36-1)11)) = \frac{36}{2} (103 + 488) = 10638$$

① ①

Simplify  $\frac{(n-1)!}{(n-4)!}$ . **NOTE: Your final answer can be in factored form.**

EITHER VERSION  
↓ IS OK

SCORE: \_\_\_\_\_ / 3 PTS

$$\frac{(n-1)(n-2)(n-3)(n-4)(n-5) \dots (3)(2)(1)}{(n-4)(n-5) \dots (3)(2)(1)} = \frac{(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} \quad \boxed{2}$$
$$= \boxed{(n-1)(n-2)(n-3)} \quad \boxed{1}$$

Consider the geometric sequence with  $a_5 = -81$  and  $a_8 = 24$ .

SCORE: \_\_\_\_ / 7 PTS

- [a] Find the formula for  $a_n$ .

$$a_8 = a_5 r^3$$

$$24 = -81r^3 \quad \textcircled{1}$$

$$r^3 = -\frac{24}{81} = -\frac{8}{27}$$

$$r = -\frac{2}{3} \quad \textcircled{1}$$

$$a_5 = a_1 r^4$$

$$-81 = a_1 \left(-\frac{2}{3}\right)^4 \quad \textcircled{1}$$

$$a_1 = (-81) \left(-\frac{3}{2}\right)^4$$

$$= \frac{-6561}{16} \quad \textcircled{1}$$

$$a_n = \frac{-6561}{16} \left(-\frac{2}{3}\right)^{n-1} \quad \textcircled{1}$$

- [b] Find the sum of the infinite series  $a_1 + a_2 + a_3 + \dots$  (based on the sequence above).

EITHER VERSION  
↓ ↓ IS OK

$$S = \frac{\frac{-6561}{16}}{1 - \left(-\frac{2}{3}\right)} \quad \textcircled{1} = \frac{-6561}{16} \cdot \frac{5}{3} = -\frac{6561}{16} \cdot \frac{3}{5} = -\frac{19683}{80} = -246.0375 \quad \textcircled{1}$$

Write  $\frac{27}{36} - \frac{23}{49} + \frac{19}{64} - \frac{15}{81} + \frac{11}{100} - \frac{7}{121} + \frac{3}{144}$  using sigma notation.

NUMERATOR: ARITHMETIC

SEQUENCE

$$d = -4$$

DENOMINATOR:  $6^2, 7^2, 8^2, \dots$

SCORE: \_\_\_\_ / 4 PTS

$$\sum_{n=1}^7 (-1)^{n+1} \frac{27 - 4(n-1)}{(n+5)^2}$$

① ②

$$\sum_{n=1}^7 (-1)^{n+1} \frac{31 - 4n}{(n+5)^2} \quad ①$$

+ ② IF INDEX OF SUMMATION (UNDER  $\Sigma$ )  
IS SAME AS INDEX IN FORMULA